A STAGE-BASED FORMULATION FOR THE FLYING SIDEKICK TRAVELING SALESMAN PROBLEM WITH JETSUITE

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Received: 13 May 2026/ Accepted: 20 July 2026/ Published: 30 July 2026

Https://doi.org/10.70117/hdujs.E11.7.2026.1372

Abstract: *This paper introduces the Flying Sidekick Traveling Salesman Problem with JetSuite (FSTSP-JS), a heterogeneous last-mile delivery problem involving a truck, a standard drone, and a human-piloted JetSuite. The drone can perform flexible launch-and-recovery sorties, whereas the JetSuite performs stationary loop sorties that require the truck to wait at the launch node. We propose a compact three-index, stage-based mixed-integer linear programming formulation to minimize the route makespan. Computational results on 180 instances with 10 customers show that FSTSP-JS reduces the average makespan by 21.3% and requires less average solution time than the classical truck-drone FSTSP baseline.*

Keywords: *FSTSP, JetSuite, truck-drone routing, last-mile delivery, mixed-integer programming.*

1. Introduction

Last-mile delivery is one of the most expensive and operationally complex stages of modern logistics. Truck-drone systems have therefore attracted considerable attention because they combine the carrying capacity of a ground vehicle with the speed and flexibility of aerial vehicles. The classical Flying Sidekick Traveling Salesman Problem (FSTSP), introduced by Murray and Chu [4], models a synchronized truck-drone system in which customers are served either by the truck or by a drone. Related variants include the Traveling Salesman Problem with Drone (TSP-D), vehicle routing problems with drones, and multi-drone extensions [1, 5-7].

***Figure 1**. Jet Suit-assisted delivery in the United Kingdom*

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Despite this progress, most existing studies assume a homogeneous fleet of autonomous drones. In practice, standard drones are limited by payload, endurance, sensing reliability, and the inability to perform deliveries requiring human authentication or delicate handling. A human-piloted JetSuite can complement standard drones in such situations. However, because of safety and operational requirements, a JetSuite sortie should be launched and recovered at the same location, and the truck should remain stationary while the JetSuite is airborne.

This paper studies the FSTSP-JS, in which a truck, a standard drone, and a JetSuite jointly serve customers. The main contribution is a compact three-index stage-based mixed-integer linear programming (MILP) formulation that simultaneously represents flexible drone sorties and stationary JetSuite loop sorties. Computational experiments compare the proposed system with the classical FSTSP baseline and evaluate the effect of aerial flight-range restrictions.

From a modeling perspective, the key difficulty is that the two aerial modes have different synchronization structures. A standard drone may be launched and recovered at different truck stages, which creates a moving rendezvous decision. In contrast, a JetSuite sortie is naturally represented as a loop because the pilot returns to the same truck location. This difference is important because it changes the feasible route architecture: the truck can act not only as a traveling vehicle, but also as a temporary service hub from which several aerial operations can be coordinated. Therefore, a stage-based representation is suitable for this problem because it records where the truck is located at each decision stage and allows aerial sorties to be attached directly to these stages.

The contribution of the paper can be summarized in three points. First, we introduce a heterogeneous extension of truck-drone delivery in which autonomous and human-piloted aerial operations are modeled simultaneously. Second, we formulate the problem as a compact stage-based MILP that distinguishes flexible drone sorties from stationary JetSuite sorties. Third, we provide a computational comparison showing that the additional stationary aerial mode can reduce makespan and may also simplify the search structure for small exact instances.

2. Problem statement and formulation

Problem description Let $C = \{1, \dots, N\}$ be the set of customers, S = 0 the starting depot, E = N + 1 the ending depot, and V = $C \cup \{S, E\}$. The truck starts at S, visits a sequence of stages, and ends at E. Each customer must be served by exactly one mode: truck, drone, or JetSuite. The standard drone may be launched from the truck at one stage, serve one customer, and be recovered at a later stage. By contrast, the JetSuite must return to the same launch node; hence the truck must use a waiting arc at that node during the JetSuite sortie. Both aerial vehicles are subject to endurance constraints. The objective is to minimize the makespan, i.e., the time at which the synchronized delivery route is completed.

The operational logic of the proposed system is illustrated by the difference between the drone and JetSuite sorties. If the truck travels from node i at stage k to node j at a later stage , the drone may be launched at i, serve customer h, and be recovered at j. This gives the drone the flexibility to support the moving truck route. By contrast, if the JetSuite is used at node i, the truck must remain at i during the flight; mathematically, this is represented by a self-loop arc x_{ii}^{k}. This modeling choice captures the safety requirement that launch and recovery take place at the same stable location.

This distinction also has a practical interpretation. A drone is suitable for light parcels and routine deliveries, while a JetSuite is more appropriate for special customers requiring direct human interaction, delivery verification, or careful handling. In dense or irregular service areas, the truck may stop at a central location and use the JetSuite to serve nearby customers without forcing the truck to visit all of them directly. As a result, the model can trade truck travel time against aerial operation time and synchronization waiting time.

Notation: The following set of notations is used to present the mathematical model of the problem.

Table 1. *Main sets, parameters, and decision variables.*

Symbol	Meaning
C, V, $\mathcal{K}$	Set of customers, set of all nodes, and set of truck stages, respectively.
C^{NV}	Set of customers that cannot be served by the standard drone.
$\tau_{ij}, \tau'_{ij}, \tau''_{ij}$	Travel times of the truck, drone, and JetSuite, respectively, from node i to node j.
D_d	Maximum aerial endurance.
t_L, t_R, tJ_L, tJ_R	Launch and retrieval service times for the drone and JetSuite.
X_i^k, x_{ij}^k	Equal 1 if the truck visits node i at stage k, or travels from i to j at stage k.
ϕ_h, ϕ_h^J	Equal 1 if customer h is served by the drone or by the JetSuite.
Y_{ih}^k, W_{hj}^k	Equal 1 if the drone is launched from i to serve h at stage k, or recovered at j after serving h at stage k.
$Z_{kk'}, Z_h^{kk'}$	Equal 1 if a drone sortie starts at stage k and ends at stage k', with or without specifying customer h.
U_{ih}^k	Equal 1 if the JetSuite is launched from node i to serve customer h at stage k and returns to i.
a_k, d_k	Arrival and departure times of the truck at stage k.

Stage-based Mixed Integer Formulation

The complete MILP formulation is presented below. It includes the core routing, synchronization, timing, strengthening, no-revisit, and domain constraints.

Objective: $\min d_K$ (1)

Customer coverage:

$$\phi_h = 0, \quad \forall h \in C^{NV}, \tag{2}$$

$$\phi_h + \phi_h^J + \sum_{k \in \mathcal{K}} X_h^k \geq 1, \quad \forall h \in C. \tag{3}$$

Truck routing and stage logic:

$$\sum_{k \in \mathcal{K}} X_h^k = 1, \qquad \forall i \in \{S, E\}, \tag{4}$$

$$X_S^1 = 1, \tag{5}$$

$$\sum_{j \in V\{S\}} x_{S_j}^1 = 1, \tag{6}$$

$$\sum_{k \in \mathcal{K}} \sum_{i \in V} x_{iE}^k = 1, \tag{7}$$

$$\sum_{k \in \mathcal{K}} \sum_{i \in V} x_{iE}^k = 0, \tag{8}$$

$$X_i^k = \sum_{j \in V\{S\}} x_{ij}^k, \qquad \forall i \in V \backslash \{ E\}, k \in \mathcal{K}, \tag{9}$$

$$X_i^k = \sum_{j \in V\{S\}} x_{ji}^{k-1}, \qquad \forall i \in V \backslash \{ E\}, k = 2, \dots, K, \tag{10}$$

$$\sum_{j \in V} x_i^k \leq 1, \qquad \forall k \in \mathcal{K}. \tag{11}$$

Drone operations and synchronization:

$$\sum_{h \in C} Y_{ih}^k \leq X_i^k, \qquad \forall i \in V, k \in \mathcal{K}, \tag{12}$$

$$\sum_{h \in C} Y_{hj}^k \leq X_j^k, \qquad \forall j \in V, k \in \mathcal{K}, \tag{13}$$

$$\sum_{i \in V} \sum_{k=1}^{K-1} Y_{ih}^k = \phi_h, \qquad \forall h \in C \tag{14}$$

$$\sum_{i \in V} \sum_{k=1}^{K-1} Y_{ih}^k = \sum_{j \in V} \sum_{k=2}^{K} Y_{hj}^k , \qquad \forall h \in C , \tag{15}$$

$$\sum_{i \in V} Y_{iE}^k = \sum_{k'=k+1}^{K} Z_h^{kk'} , \qquad \forall h \in C, k \in \mathcal{K}, \tag{16}$$

$$\sum_{i \in V} W_{hj}^{k'} = \sum_{k=1}^{k'-1} Z_h^{kk'} , \qquad \forall h \in C,\ k' \in \mathcal{K}, \tag{17}$$

$$Z_{kk'} = \sum_{h \in C} Z_h^{kk'}, \qquad \forall\ k < k', \tag{18}$$

$$\sum_{k=1}^{m} \sum_{k'=m+1}^{K} Z_{kk'} \leq 1, \forall m = 1, \dots., K-1. \tag{19}$$

JetSuite operations:

$$\sum_{h\in C} U_{ih}^{k} \le x_{ii}^{k}, \qquad \forall i \in V, k \in \mathcal{K}, \tag{20}$$

$$\sum_{i\in V}\sum_{k=1}^{K-1} U_{ih}^{k} = \phi_h^J, \qquad \forall h \in C. \tag{21}$$

Time propagation and endurance:

$$a_1 = 0,\ d_1 = 0, \tag{22}$$

$$d_k \ge a_k + \sum_{k'=1}^{k-1} Z_{kk'} t_R + \sum_{k'=k+1}^{K} Z_{kk'} t_L\ , \qquad \forall\, k \in \mathcal{K}, \tag{23}$$

$$a_{k+1} = \ d_k + \sum_{i\in V}\sum_{j\in V} x_{ij}^{k}\tau_{ij}\,, \qquad \forall k = 1, \dots K-1, \tag{24}$$

$$a_{k'} \le d_k + Z_{kk'}(D_d - t_R) + M(1 - Z_{kk'}), \quad \forall\, k < k', \tag{25}$$

$$\sum_{k\in\mathcal{K}}\sum_{i\in V} Y_{ih}^{k}\tau'_{ih} + \sum_{k\in\mathcal{K}}\sum_{i\in V} W_{hj}^{k}\tau'_{hj} \le (D_d - t_R)\phi_h, \qquad \forall h \in C\,, \tag{26}$$

$$\sum_{k=1}^{K-1}\sum_{i\in V} U_{ih}^{k}(\tau''_{ih} + \tau''_{hi}) \le (D_d - tJ_R)\phi_h^J, \qquad \forall h \in C, \tag{27}$$

$$\begin{aligned} d_{k'} + M(1 - Z_{kk'}) \\ &\ge d_k \\ &+ \sum_{i\in V}\sum_{h\in C} W_{hi}^{k'}\tau'_{hj} + \ Z_{kk'} t_R \\ &+ \sum_{\bar{k}=k'+1}^{K} \ Z_{k'\bar{k}} t_L, \qquad \forall k < k', \end{aligned} \tag{28}$$

$$\begin{aligned} d_{k+1} + M(1 - \sum_{i\in V}\sum_{h\in C} U_{ih}^{k}) \\ &\ge d_k \\ &+ \sum_{i\in V}\sum_{h\in C} U_{ih}^{k}\,(\,\tau''_{ih} + \tau''_{hi} + tJ_R) \\ &+ \sum_{i\in V}\sum_{h\in C} U_{ih}^{k+1}\, tJ_R, \qquad \forall k = 1, \dots K-1. \end{aligned} \tag{29}$$

Strengthening constraints and valid inequalities:

$$a_1 = 0,\ d_1 = 0, \tag{30}$$

$$d_k \ge a_k + \sum_{k'=1}^{k-1} Z_{kk'} t_R + \sum_{k'=k+1}^{K} Z_{kk'} t_L\ , \qquad \forall\, k \in \mathcal{K}, \tag{31}$$

$$a_{k+1} = \ d_k + \sum_{i\in V}\sum_{j\in V} x_{ij}^{k}\tau_{ij}\,, \qquad \forall k = 1, \dots K-1, \tag{32}$$

$$a_{k'} \le d_k + Z_{kk'}(D_d - t_R) + M(1 - Z_{kk'}), \quad \forall\, k < k', \tag{33}$$

$$\sum_{k\in\mathcal{K}}\sum_{i\in V} Y_{ih}^{k}\tau'_{ih} + \sum_{k\in\mathcal{K}}\sum_{i\in V} W_{hj}^{k}\tau'_{hj} \le (D_d - t_R)\phi_h, \qquad \forall h \in C\,, \tag{34}$$

$$\sum_{k=1}^{K-1}\sum_{i\in V} U_{ih}^{k}(\tau_{ih}'' + \tau_{hi}'') \le (D_d - tJ_R)\phi_h^J, \qquad \forall h \in C, \tag{35}$$

$$d_{k'} + M(1 - Z_{kk'}) \ge d_k + \sum_{i\in V}\sum_{h\in C} W_{hi}^{k'}\tau_{hj}' + Z_{kk'}t_R + \sum_{\bar{k}=k'+1}^{K} Z_{k'\bar{k}}t_L, \qquad \forall k < k', \tag{36}$$

$$d_{k+1} + M(1 - \sum_{i\in V}\sum_{h\in C} U_{ih}^{k}) \ge d_k + \sum_{i\in V}\sum_{h\in C} U_{ih}^{k}\,(\tau_{ih}'' + \tau_{hi}'' + tJ_R) + \sum_{i\in V}\sum_{h\in C} U_{ih}^{k+1}\, tJ_R, \qquad \forall k = 1, \dots K-1. \tag{37}$$

$$\sum_{i\in V}\sum_{p=1}^{k} Y_{ih}^{p} \ge \sum_{i\in V}\sum_{p=1}^{k+1} W_{hj}^{p}, \qquad \forall h \in C, k < K. \tag{38}$$

No-revisit rule and domains:

$\sum_{k=1}^{K-1}\sum_{i\in V\{i\}} x_{ij}^{k} \le 1, \qquad \forall i \in V,$	(39)
$X_i^k, x_{ij}^k, \phi_h, \phi_h^J, Y_{ih}^k, W_{hj}^k, Z_{kk'}, Z_h^{kk'}, U_{ih}^k \in \{0,1\}, \quad \forall\, i,j \in V, h \in C, k,k' \in \mathcal{K},$	(40)
$Z_{kk'} = 0, Z_h^{kk'} = 0, \quad \forall h \in C, k' \le k,$	(41)
$x_{EE}^k = 0, x_{SS}^k = 0, \quad \forall k \in \mathcal{K},$	(42)
$Y_{ih}^k = 0, W_{hi}^k = 0, \quad \forall h \in C, i = h, k \in \mathcal{K},$	(43)
$a_k \ge 0,\ d_k \ge 0, \qquad \forall k \in \mathcal{K}.$	(44)

The first four blocks define service assignment, truck movement, flexible drone sorties, and stationary JetSuite sorties. The time block links routing choices with departure and arrival times, while the remaining constraints tighten the formulation, rule out revisits when required, and define variable domains.

3. Experiment and Discussion

The experiments were conducted on 180 randomly generated Euclidean instances with ten customers. Customer locations were uniformly distributed in a two-dimensional area. All MILP models were solved to optimality using CPLEX 22.1.1.0 on a workstation with an Intel Core i5-14500 processor and 64 GB RAM. The proposed FSTSP-JS formulation was compared with the classical FSTSP model without the JetSuite.

Table 2. *Performance comparison between FSTSP-JS and the classical FSTSP baseline.*

Metric	FSTSP-JS	FSTSP
Valid instances	180	180
Minimum objective	97.86	148.52
Maximum objective	338.06	351.94
Average objective	211.27	268.56
Average optimality gap (%)	0.01	0.00
Average solution time (s)	23.55	56.89

Table 2 shows that introducing the JetSuite substantially improves both route quality and computational performance. The average objective value decreases from 268.56 to 211.27, corresponding to a 21.3% reduction in makespan. The average solution time also decreases from 56.89 seconds to 23.55 seconds. This suggests that stationary aerial sorties can offload distant or inconvenient customers from the truck route while preserving a compact synchronization structure.

The improvement can be explained by the hub effect created by the JetSuite. In the classical FSTSP, the truck must still travel to many customers that are not attractive for the drone because of range or synchronization restrictions. In FSTSP-JS, a truck stop can be used as a stationary launch point for the JetSuite, allowing the truck to avoid detours to some peripheral customers. This is particularly useful when customers form local clusters around a few potential stopping points. The lower average solution time also suggests that the JetSuite does not simply add complexity; it may create alternative service patterns that make high-quality solutions easier to identify for small instances.

However, the result should be interpreted carefully. The experiments are based on small exact instances with 10 customers, and therefore they mainly validate the correctness and potential benefit of the proposed formulation. For larger instances, exact MILP solution may become computationally expensive. In that case, the compact model can still be used as a component of a decomposition method, a local search neighborhood, or a matheuristic framework.

Table 3 further indicates that the proposed model benefits from larger aerial operating ranges. When the maximum radius increases from 20% to 100%, the FSTSP-JS objective decreases from 291.445 to 227.860. The classical FSTSP baseline also improves when the radius is relaxed, but its objective values remain worse than those of FSTSP-JS in all tested cases. These results support the practical value of combining flexible drone sorties with stationary JetSuite sorties.

Table 3. *Impact of maximum flight radius on one representative instance*

Radius (%)	FSTSP-JS			FSTSP		
	Obj.	Time	Gap	Obj.	Time	Gap
20	291.445	1.700	0.00	300.042	0.279	0.00
40	274.456	35.335	0.00	300.042	0.762	0.00
60	251.687	48.186	0.00	291.817	1.968	0.00
100	227.860	19.699	0.01	287.226	41.360	0.00
150	227.860	18.321	0.00	251.687	46.729	0.01

The sensitivity analysis also reveals a non-monotonic pattern in solution time. When the aerial radius is very small, only a limited number of sorties is feasible, and the solver can quickly eliminate infeasible combinations. When the radius becomes moderate, many more launch-and-recovery combinations become feasible, increasing the number of alternatives that must be evaluated. When the radius is sufficiently large, the model has more flexibility, but many dominated choices can be pruned by the objective and by time consistency. This explains why the largest radius does not necessarily produce the largest solution time.

From a managerial viewpoint, the results indicate that the value of the JetSuite is not only related to speed. Its main benefit comes from changing the structure of the delivery plan. Instead of forcing the truck to visit every difficult customer, the system can select a smaller number of useful stopping points and serve surrounding customers by aerial modes. This suggests that JetSuite deployment should be planned together with the location of safe launch points, expected customer density, and the operational cost of truck waiting.

The proposed model also highlights several implementation issues. First, a real delivery operator must define which customers are eligible for drone delivery and which require human-piloted service. Second, endurance should be adjusted according to payload, weather, and safety margins. Third, the waiting time of the truck should be evaluated against the time saved by avoiding ground detours. These issues do not change the mathematical structure of the formulation, but they determine how the model should be parameterized in real applications.

4. Conclusion

This paper proposed the FSTSP-JS, a heterogeneous extension of truck-drone delivery that combines a truck, a standard drone, and a human-piloted JetSuite. A compact three-index stage-based MILP formulation was developed to represent both flexible drone sorties and stationary JetSuite loop sorties. Computational results show that the JetSuite-enabled system reduces the average makespan by 21.3% and requires less average solution time than the classical FSTSP baseline. Future research should develop scalable metaheuristics for larger instances and incorporate more detailed energy, payload, and environmental constraints.

The study contributes to the literature by showing that heterogeneous aerial capabilities can be represented within a single stage-based optimization model. In particular, the formulation separates two synchronization mechanisms: moving rendezvous for the standard drone and stationary loop operation for the JetSuite. This distinction is useful for future studies on mixed fleets, special-service customers, and delivery systems that combine autonomous vehicles with human-operated aerial devices.

References

[1] Agatz, N., Bouman, P., and Schmidt, M. (2018), *Optimization approaches for the traveling salesman problem with drone*. Transportation Science, 52(4), 965-981, 2018.

[2] Boccia, M., Masone, A., Sforza, A., and Sterle, C.(2023), *A new arc-based formulation and branch-and-cut for the flying sidekick traveling salesman problem*. Transportation Science.

[3] Dell'Amico, M., Montemanni, R., and Novellani, S. (2019), *Models and algorithms for the flying sidekick traveling salesman problem*. Optimization Letters, 13, 1613-1641.

[4] Murray, C. C., and Chu, A. G.(2015), *The flying sidekick traveling salesman problem: Optimization of drone-assisted parcel delivery*. Transportation Research Part C, 54, 86-109.

[5] Roberti, R., and Ruthmair, M.(2021), *Exact methods for the traveling salesman problem with drone*. Transportation Science, 55(2), 315-335.

[6] Sacramento, D., Pisinger, D., and Ropke, S. (2019), *An adaptive large neighborhood search metaheuristic for the vehicle routing problem with drones*. Transportation Research Part C, 102, 289-315.

[7] Wang, K., Yuan, B., Zhao, M., and Lu, Y. (2022), *Cooperative truck and drone delivery with time-dependent road travel*. Transportation Research Part C.