HYERS-ULAM STABILITY, WELL-POSEDNESS, AND DATA DEPENDENCE FOR FISHER-TYPE CONTRACTIVE MAPPINGS

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Abstract: *In this paper, we study Hyers–Ulam stability, well-posedness, and data dependence for mappings satisfying a Fisher-type inequality. Assuming that the mapping* T *has a fixed point* z*, we first show that the Fisher-type inequality implies the unique-preimage property* $T^{-1}(\{z\}) = \{z\}$*. This structural property yields the global residual estimate* $d(x,z) \le \frac{1}{1-c} d(x,Tx)$, $x \in X$*, where* $c \in [0,1)$ *is the Fisher parameter. Consequently, Hyers–Ulam stability and well-posedness follow in an arbitrary metric space, without compactness, completeness, or continuity assumptions. Data dependence is treated separately: if a perturbed mapping* S *has a fixed point* w *and satisfies* $d(Tx,Sx) \le \eta$ *for all* $x \in X$*, then* $d(z,w) \le \eta/(1-c)$*. Combining these abstract estimates with the fixed point theorems in [10], we obtain existence, uniqueness, convergence, and stability conclusions in boundedly compact and* T*-orbitally compact metric spaces, and in complete metric spaces under an additional asymptotic condition. Finally, we present an example of a discontinuous mapping, together with numerical illustrations.*

Keywords: *Fixed point; Fisher-type contraction; well-posedness; Hyers–Ulam stability; data dependence.*

1. Introduction

The Banach contraction principle is a standard tool for solving the fixed point equation $Tx = x$ in metric spaces. A large part of the subsequent literature concerns contractive conditions that differ from the Lipschitz form, including Kannan-, Chatterjea-, and Ćirić-type inequalities (see [12], [4], [5]), as well as extensions to generalized metric settings.

In [7], Fisher considered mappings on compact metric spaces satisfying the inequality

$$d\left(Tx,Ty\right)^2 < d\left(x,Tx\right)d\left(y,Ty\right) + c\,d\left(x,Ty\right)d\left(y,Tx\right),\ x \ne y, \tag{1}$$

for some $c \in \left[0,1\right)$. In [10], Hoc et al. extended Fisher's fixed point theorem by establishing existence and uniqueness of fixed points for Fisher-type mappings under

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significantly weaker assumptions: they replaced the compactness of the space with weaker topological conditions such as completeness, bounded compactness, or T-orbital compactness, and relaxed the continuity of T to orbital continuity.

The present paper studies several stability aspects of fixed point problems for mappings satisfying the Fisher-type inequality (1). In particular, we consider three related notions: Hyers-Ulam stability, well-posedness, and data dependence. Our main objective is to develop general techniques for establishing these properties within the Fisher-type framework.

Our approach is based on a simple structural consequence of the Fisher-type inequality. We show that if z is a fixed point of T, then z has no preimage other than itself, i.e. $T^{-1}(\{z\}) = \{z\}$. This unique-preimage property is the key point that yields the global residual estimate used throughout the paper whenever a fixed point exists. We then apply the general results to the metric space classes treated in [10], thereby obtaining complete statements combining existence and stability. Finally, we present an example of a discontinuous mapping showing that the conclusions do not rely on the global continuity of T.

Stability properties of fixed point problems play an important role in applications, especially when exact solutions are difficult to compute or when the underlying data are subject to perturbations. Hyers–Ulam stability ensures that approximate solutions generated by numerical algorithms or measurement errors remain close to true fixed points. Well-posedness guarantees robustness of the fixed point problem with respect to small perturbations, while data dependence provides explicit quantitative estimates describing how fixed points vary under changes of the operator. Such properties are relevant in numerical analysis, optimization, iterative methods, and the study of nonlinear functional and integral equations.

The quantitative core of the paper is the following global residual estimate:

$$d(x,z) \le \frac{1}{1-c} d(x,Tx), \quad x \in X,$$

where z is the fixed point of T. This estimate directly yields Hyers–Ulam stability and well-posedness. Applied to a fixed point w of a perturbed mapping S satisfying $d(Tx,Sx) \le \eta$ for all $x \in X$, it also gives the data-dependence bound $d(z,w) \le \eta/(1-c)$

2. Preliminaries

We begin by recalling the basic notions related to fixed point problems.

Definition 2.1 ([9]). Let (X,d) be a metric space and $T: X \to X$ be a mapping. A point $z \in X$ is called a *fixed point* of T if $Tz = z.$

The set of all fixed points of T is denoted by $\mathrm{Fix}(T) = \{x \in X : Tx = x\}.$

The problem of finding a fixed point of T is called the *fixed point problem* associated with T.

Definition 2.2 ([6]). A metric space (X,d) is said to be *boundedly compact* if every bounded sequence in X has a convergent subsequence.

It is easy to see that every compact metric space is boundedly compact, but the converse is not true. For instance, the Euclidean space R^n with the usual metric is boundedly compact (every bounded sequence has a convergent subsequence by the Bolzano–Weierstrass theorem), but it is not compact.

Definition 2.3 ([5]). Let (X, d) be a metric space and T be a self-mapping on X. The orbit of T at $x \in X$ is defined as $O_x(T) = \{x, Tx, T^2x, T^3x, \ldots\}$.

Definition 2.4 ([8]). Let (X, d) be a metric space and $T : X \to X$. The space X is said to be T*-orbitally compact* if every sequence in the orbit $O_x(T) = \{x, Tx, T^2x, \ldots\}$ has a convergent subsequence for all $x \in X$.

Definition 2.5 ([2]). Let (X, d) be a metric space and $T : X \to X$ be a self-mapping. T is said to be orbitally continuous at a point z in X if for any sequence $\{x_n\} \subseteq O_x(T)$ for some $x \in X, x_n \to z$ as $n \to \infty$ implies $Tx_n \to Tz$ as $n \to \infty$.

We now recall the stability concepts which are the focus of this paper.

Definition 2.6 ([15]). Let (X, d) be a metric space and $T : X \to X$ be a mapping. The fixed point problem for T is said to be *well-posed* if:

T has a unique fixed point $z \in X$;

For any sequence $\{x_n\} \subset X$ such that $\lim_{n\to\infty} d(x_n, Tx_n) = 0$, we have $\lim_{n\to\infty} x_n = z$.

Remark 2.7. This concept was introduced by Reich and Zaslavski [15] and further developed for multivalued operators in [13, 14].

Definition 2.8 ([3]). Let (X, d) be a metric space. Let $T : X \to X$ be an operator having a unique fixed point z, and let $S : X \to X$ be another operator with $\mathrm{Fix}(S) \neq \varnothing$. The fixed point problem is said to have the *data dependence property* if there exists a constant $K > 0$ such that

$$d(z, w) \leq K \sup_{x\in X} d(Tx, Sx)$$

for every $w \in \mathrm{Fix}(S)$.

Finally, we recall the concept of Hyers–Ulam stability. The concept of stability for functional equations originated from a question raised by Ulam [18] in 1940, concerning the approximate homomorphisms of groups. In the following year, Hyers [11] provided the first affirmative answer to Ulam's question for additive mappings on Banach spaces. The connection between this stability concept and fixed point theory has been extensively studied by Rus [16, 17]. In particular, we adopt the formal definition provided in [17, Definition 2.2] and use the term Hyers–Ulam stability throughout this paper.

Definition 2.9. Let (X, d) be a metric space and let $T : X \to X$ be an operator. The fixed point equation $x = Tx$ is said to be Hyers–Ulam stable with constant $K_T > 0$ if, for every $\varepsilon > 0$ and each $x \in X$ satisfying

$$d(x, Tx) \leq \varepsilon, \tag{1}$$

there exists a solution $z \in \mathrm{Fix}(T)$ such that

$$d(x,z) \le K_T \varepsilon. \tag{2}$$

Definition 2.10 ([1]). Let (X,d) be a metric space and let $T: X \to X$ be a mapping. For $x_0 \in X$, the sequence $\{x_n\}$ defined by

$$x_n = Tx_{n-1},\ n \ge 1,$$

is called the Picard iteration sequence with initial point x_0.

The mapping T is said to be a Picard operator if it has a unique fixed point and every Picard iteration in X converges to the fixed point.

Remark 2.11. In the terminology of Rus [16, 17], an operator T is called a *weakly Picard operator* if, for every $x \in X$, the sequence of successive approximations $(T^n x)$ converges to a fixed point of T. A weakly Picard operator having a unique fixed point is called a *Picard operator*. Moreover, if a Picard operator with unique fixed point z satisfies $d(x,z) \le Cd(x,Tx)$ for all $x \in X$, then it is a C*-Picard operator*. According to [17], every C-Picard operator is Hyers–Ulam stable.

3. Results

In this section, we establish Hyers–Ulam stability, well-posedness, and data dependence for Fisher-type mappings in a general metric space setting. Theorems 3.3 and 3.4 use only the Fisher-type inequality and the existence of a fixed point; no compactness, completeness, or continuity assumptions are required. The data-dependence result additionally assumes that the perturbed mapping has a fixed point and is uniformly close to the original mapping.

Lemma 3.1. Let (X,d) be a metric space and let $T: X \to X$ satisfy (1) for some $c \in [0,1)$. Then:

(i). T has at most one fixed point.

(ii). If z is a fixed point of T, then $T^{-1}(\{z\}) = \{z\}$; equivalently, $y \ne z \Rightarrow Ty \ne z$

Proof. (i) Suppose $z_1 \ne z_2$ and $Tz_1 = z_1$, $Tz_2 = z_2$. Applying (1) to (z_1, z_2) yields

$$d(z_1,z_2)^2 = d(Tz_1,Tz_2)^2 < c\,d(z_1,z_2)^2,$$

hence $(1-c)d(z_1,z_2)^2 < 0$, a contradiction.

(ii) Let $Tz = z$ and assume $y \ne z$ with $Ty = z$. Then $Ty = Tz$ while $y \ne z$. Applying (1) to (y,z) gives

$$0 = d(Ty,Tz)^2 < d(y,Ty)\,d(z,Tz) + c\,d(y,Tz)\,d(z,Ty) = 0,$$

a contradiction. Thus $Ty \ne z$ for all $y \ne z$.

Remark 3.2. The conclusion of Lemma 3.1(ii) is a structural feature specific to the Fisher-type inequality (1). It guarantees that $d\left(Ty,z\right)>0$ whenever $y\neq z$, which is essential for dividing by $d\left(Ty,z\right)$ in the stability-type estimates.

We first establish Hyers–Ulam stability for Fisher-type mappings.

Theorem 3.3. Let (X,d) be a metric space and let $T:X\to X$ satisfy (1) with some $c\in[0,1)$. Assume that T has a fixed point z. Then the fixed point equation $x=Tx$ is Hyers–Ulam stable with the constant

$$K_T=\frac{1}{1-c}.$$

More precisely, the following global residual estimate holds for every $y\in X$:

$$d(y,z)\le\frac{1}{1-c}d(y,Ty),\quad y\in X.$$

Proof. If $c=0$ and X contains a point $y\neq z$, then applying (1) to (z,y) gives $d(z,Ty)^2<0$, which is impossible. Hence $X=\{z\}$, and the conclusion is immediate. Assume now that $0<c<1$, and let $y\in X$. If $y=z$, the estimate is trivial. Assume $y\neq z$. By Lemma 3.1(ii), $Ty\neq z$, hence $d(z,Ty)>0$. Applying (1) to $x=z$ and y yields

$$d(z,Ty)^2<d(z,Tz)d(y,Ty)+cd(z,Ty)d(y,Tz)=cd(z,Ty)d(y,z).$$

Since $d(z,Ty)>0$, division gives

$$d(z,Ty)<cd(y,z).$$

Using the triangle inequality,

$$d(y,z)\le d(y,Ty)+d(Ty,z)<d(y,Ty)+cd(y,z).$$

Hence

$$(1-c)d(y,z)\le d(y,Ty),$$

and therefore

$$d(y,z)\le\frac{1}{1-c}d(y,Ty).$$

If $d(y,Ty)\le\varepsilon$, the last estimate gives $d(y,z)\le\varepsilon/(1-c)$. Therefore, the fixed point equation is Hyers–Ulam stable with constant $K_T=1/(1-c)$.

Next, we address the well-posedness of the fixed point problem.

Theorem 3.4. Under the assumptions of Theorem 3.3, the fixed point problem for T is well-posed. In particular, if $\{u_n\}\subset X$ satisfies $d(u_n,Tu_n)\to 0$, then $u_n\to z$ as $n\to\infty$.

Proof. By Lemma 3.1(i), z is the unique fixed point of T. Applying Theorem 3.3 with $y=u_n$, we obtain

$$d(u_n,z)\le\frac{1}{1-c}d(u_n,Tu_n).$$

Taking the limit as $n \to \infty$, we obtain $d(u_n, z) \to 0$, i.e., $u_n \to z$ as $n \to \infty$. Thus, the fixed point problem for T is well-posed.

Finally, we investigate the data dependence of fixed points.

Theorem 3.5. Let (X,d) be a metric space and let $T : X \to X$ satisfy (1) with some $c \in [0,1)$. Assume that T has a fixed point z. Let $S : X \to X$ be another mapping having a fixed point w. If there exists $\eta \geq 0$ such that

$$d(Tx, Sx) \leq \eta, \quad x \in X,$$

Then

$$d(z,w) \leq \frac{\eta}{1-c}.$$

Proof. Since $Sw = w$, the uniform closeness assumption, applied at $x = w$, gives

$$d(w,Tw) = d(Sw,Tw) \leq \eta.$$

Applying the global residual estimate in Theorem 3.3 with $y = w$, we obtain

$$d(z,w) = d(w,z) \leq \frac{1}{1-c} d(w,Tw) \leq \frac{\eta}{1-c}.$$

We now illustrate how the abstract stability results obtained above can be combined with existing fixed point existence theorems to derive complete conclusions, including existence, uniqueness, convergence of Picard iterations, and stability properties, in several important classes of metric spaces from [10].

As a first application, we consider boundedly compact metric spaces, which extend the classical compact setting originally studied by Fisher.

Corollary 3.6. Let (X,d) be a boundedly compact metric space and let $T : X \to X$ be orbitally continuous. Assume that (1) holds for some $c \in [0,1)$. Then T has a unique fixed point $z \in X$ and, for every $x \in X$, the Picard iteration $(T^n x)$ converges to z. Moreover, the fixed point equation for T is Hyers–Ulam stable with constant $K_T = 1/(1-c)$, and the fixed point problem is well-posed. In addition, if $S : X \to X$ has a fixed point w and there exists $\eta \geq 0$ such that $d(Tx, Sx) \leq \eta$ for all $x \in X$, then $d(z,w) \leq \eta/(1-c)$.

Proof. By [10, Theorem 2.2], T is a Picard operator; hence it has a unique fixed point z and $T^n x \to z$ for all $x \in X$. Hyers–Ulam stability and well-posedness follow from Theorems 3.3 and 3.4, respectively, and the final data-dependence estimate follows from Theorem 3.5.

The next result addresses the case of T-orbitally compact metric spaces, which allows the compactness assumption to be restricted to the orbits of the mapping.

Corollary 3.7. Let (X,d) be a T-orbitally compact metric space and let $T : X \to X$ be orbitally continuous. Assume that (1) holds for some $c \in [0,1)$. Then T

has a unique fixed point $z \in X$ and, for every $x \in X$, the Picard iteration $(T^n x)$ converges to z. Moreover, the fixed point equation for T is Hyers–Ulam stable with constant $K_T = 1/(1-c)$, and the fixed point problem is well-posed. In addition, if $S : X \to X$ has a fixed point w and there exists $\eta \geq 0$ such that $d(Tx, Sx) \leq \eta$ for all $x \in X$, then $d(z, w) \leq \eta/(1-c)$.

Proof. The existence and uniqueness of the fixed point, as well as the convergence of $T^n x$ for all $x \in X$, follow from [10, Theorem 2.3]. Hyers–Ulam stability and well-posedness follow from Theorems 3.3 and 3.4, respectively, and the final data-dependence estimate follows from Theorem 3.5.

Finally, we apply the general framework to complete metric spaces under an additional asymptotic regularity condition, recovering convergence and stability of the Picard iteration.

Corollary 3.8. Let (X, d) be a complete metric space and let $T : X \to X$ satisfy (1) with some $c \in [0,1)$. Assume further that, for every $x \in X$ and every $\varepsilon > 0$, there exists $\delta > 0$ such that

$$d\left(T^i x, T^j x\right) < \varepsilon + \delta \Rightarrow d\left(T^{i+1} x, T^{j+1} x\right) \leq \varepsilon, \forall i, j \in \mathrm{N} \cup \{0\}.$$

Then T has a unique fixed point $z \in X$ and, for every $x \in X$, the Picard iteration $(T^n x)$ converges to z. Moreover, the fixed point equation for T is Hyers–Ulam stable with constant $K_T = 1/(1-c)$, and the fixed point problem is well-posed. In addition, if $S : X \to X$ has a fixed point w and there exists $\eta \geq 0$ such that $d(Tx, Sx) \leq \eta$ for all $x \in X$, then $d(z, w) \leq \eta/(1-c)$.

Proof. The Picard property follows from [10, Theorem 2.4]. Hyers–Ulam stability and well-posedness follow from Theorems 3.3 and 3.4, respectively, and the final data-dependence estimate follows from Theorem 3.5.

4. Example and Numerical Illustration

In this section we present a concrete example of a discontinuous mapping satisfying the Fisher-type condition. In addition to verifying the contractive inequality, we provide numerical illustrations of the Hyers–Ulam stability, well-posedness, and data dependence results established in the preceding section.

Example 4.1. Let $X = [0,1]$ endowed with $d(x, y) = |x - y|$. Define $T : X \to X$ by

$$T(x) = \begin{cases} \dfrac{x}{5}, & x \in [0,1), \\ \dfrac{1}{3}, & x = 1. \end{cases}$$

Then $z = 0$ is the unique fixed point of T, and T is discontinuous at $x = 1$.

Verification of the Fisher-type condition. We show that T satisfies (1) with $c=0.8=\frac{4}{5}$. Since $d(u,v)=|u-v|$, it suffices to verify that for all distinct $x,y\in[0,1]$,

$$|T(x)-T(y)|^2<|x-Tx||y-Ty|+c|x-Ty||y-Tx|. \tag{3}$$

Case 1: $x,y\in[0,1)$**.** Assume $x>y\geq 0$ and set $q=y/x\in[0,1)$. A direct computation gives

$$\text{LHS}=\left|\frac{x}{5}-\frac{y}{5}\right|^2=\frac{x^2}{25}(1-q)^2\leq\frac{x^2}{25}=0.04\,x^2.$$

Moreover,

$$|x-Tx|=\frac{4x}{5},\quad |y-Ty|=\frac{4y}{5}=\frac{4qx}{5},$$

$$|x-Ty|=x\left(1-\frac{q}{5}\right),\quad |y-Tx|=x\left|q-\frac{1}{5}\right|.$$

Hence

$$\text{RHS}=x^2\left(\frac{16}{25}q+c\left(1-\frac{q}{5}\right)\left|q-\frac{1}{5}\right|\right).$$

If $0\leq q\leq\frac{1}{5}$, then $\left|q-\frac{1}{5}\right|=\frac{1}{5}-q$ and $1-\frac{q}{5}\geq\frac{24}{25}$, so

$$\text{RHS}\geq x^2\left(\frac{16}{25}q+\frac{4}{5}\cdot\frac{24}{25}\left(\frac{1}{5}-q\right)\right)\geq x^2\cdot\frac{16}{125}=0.128\,x^2.$$

If $\frac{1}{5}\leq q<1$, then $\left|q-\frac{1}{5}\right|=q-\frac{1}{5}$ and $1-\frac{q}{5}\geq\frac{4}{5}$, so

$$\text{RHS}\geq x^2\left(\frac{16}{25}q+\frac{4}{5}\cdot\frac{4}{5}\left(q-\frac{1}{5}\right)\right)\geq x^2\cdot\frac{16}{125}=0.128\,x^2.$$

Thus $\text{LHS}\leq 0.04x^2<0.128x^2\leq\text{RHS}$, and (4) holds in Case 1.

Case 2: one of x,y **equals** 1**.** Assume $x=1$ and $y\in[0,1)$. Then $T(1)=\frac{1}{3}$ and $T(y)=\frac{y}{5}$, so

$$\text{LHS}=\left|\frac{1}{3}-\frac{y}{5}\right|^2\leq\left(\frac{1}{3}\right)^2=\frac{1}{9}\approx 0.111\ldots$$

Also $|x-Tx|=\frac{2}{3}$ and $|y-Ty|=\frac{4y}{5}$, hence

$$\text{RHS}=\frac{2}{3}\cdot\frac{4y}{5}+c\left(1-\frac{y}{5}\right)\left|y-\frac{1}{3}\right|=\frac{8y}{15}+\frac{4}{5}\left(1-\frac{y}{5}\right)\left|y-\frac{1}{3}\right|.$$

If $0 \le y \le \frac{1}{3}$, then $\left|y-\frac{1}{3}\right| = \frac{1}{3} - y$ and $1-\frac{y}{5} \ge \frac{14}{15}$, so

$$\text{RHS} \ge \frac{8y}{15} + \frac{4}{5}\cdot\frac{14}{15}\left(\frac{1}{3} - y\right),$$

whose minimum on $\left[0, \frac{1}{3}\right]$ is attained at $y = \frac{1}{3}$, giving $\text{RHS} \ge \frac{8}{45} \approx 0.177\ldots.$

If $\frac{1}{3} < y < 1$, then $1-\frac{y}{5} \ge \frac{4}{5}$ and $\left|y-\frac{1}{3}\right| = y - \frac{1}{3}$, hence

$$\text{RHS} \ge \frac{8y}{15} + \frac{4}{5}\cdot\frac{4}{5}\left(y - \frac{1}{3}\right),$$

which is increasing in y and thus $\text{RHS} \ge \frac{8}{45} \approx 0.177\ldots$ as well. Therefore

$$\text{LHS} \le \frac{1}{9} < \frac{8}{45} \le \text{RHS},$$

and (4) holds in Case 2.

Combining the two cases, T satisfies (1) with $c = 0.8$. Consequently, the stability constant in Theorem 3.3 is

$$K_T = \frac{1}{1-c} = \frac{1}{1-0.8} = 5.$$

Illustration of Hyers–Ulam stability. Let $\varepsilon = 0.1$ and take $y = 0.1$. Then $Ty = \frac{y}{5} = 0.02$, so

$$d\left(y, Ty\right) = \left|0.1 - 0.02\right| = 0.08 \le 0.1 = \varepsilon.$$

Theorem 3.3 yields $d\left(y, z\right) \le K_T\varepsilon = 0.5$, while here

$$d\left(y, z\right) = \left|0.1 - 0\right| = 0.1 \le 0.5.$$

Illustration of well-posedness. Consider $x_n = \frac{1}{n}$ for $n \ge 2$, so $x_n \in \left[0,1\right)$ and $Tx_n = \frac{1}{5n}$. Then

$$d\left(x_n, Tx_n\right) = \left|\frac{1}{n} - \frac{1}{5n}\right| = \frac{4}{5n} \to 0,$$

and $x_n \to 0 = z$, which is consistent with Theorem 3.4.

Illustration of data dependence. Define $S : X \to X$ by

$$S\left(x\right) = \begin{cases} \frac{x}{5} + 0.01, & x \in \left[0,1\right), \\ \frac{1}{3} + 0.01, & x = 1. \end{cases}$$

Then $S(X) \subset X$ and $|Tx - Sx| = 0.01$ for all $x \in X$, so one may take $\eta = 0.01$ in Theorem 3.5. The fixed point w of S in $[0,1)$ solves

$$w = \frac{w}{5} + 0.01 \Rightarrow w = \frac{0.01}{1 - \frac{1}{5}} = 0.0125.$$

Hence

$$d(z,w) = 0.0125 \le 0.05 = \frac{\eta}{1-c},$$

as predicted by Theorem 3.5.

Numerical illustration of Picard iteration. To further illustrate the convergence of the Picard iteration $x_{n+1} = T(x_n)$, we list several iterates for different initial points. In both cases below, the iterates converge to the fixed point $z = 0$. For $x_0 \in [0,1)$ one has the explicit formula $x_n = \frac{x_0}{5^n}$. For $x_0 = 1$, the first step gives $x_1 = T(1) = \frac{1}{3}$, and thereafter $x_{n+1} = \frac{x_n}{5}$ for $n \ge 1$, hence $x_n = \frac{1}{3 \cdot 5^{n-1}}$ for $n \ge 1$. We also report the error $e_n = d(x_n, 0) = |x_n|$, its decimal logarithm, and the ratio $r_n := e_n / e_{n-1}$ (for $n \ge 1$).

Table 1. ***Picard iterates $x_{n+1} = T(x_n)$, errors $e_n = |x_n|$, and ratios $r_n = e_n/e_{n-1}$.***

	Start $x_0 = 1$				**Start $x_0 = 0.5$**			
n	**x_n**	**e_n**	**log(e_n)**	**r_n**	**x_n**	**e_n**	**log(e_n)**	**r_n**
0	1.000000	1.000000	0.000000	–	0.500000	0.500000	-0.301030	–
1	0.333333	0.333333	-0.477121	0.333333	0.100000	0.100000	-1.000000	0.200000
2	0.066667	0.066667	-1.176091	0.200000	0.020000	0.020000	-1.698970	0.200000
3	0.013333	0.013333	-1.875061	0.200000	0.004000	0.004000	-2.397940	0.200000
4	0.002667	0.002667	-2.574031	0.200000	0.000800	0.000800	-3.096910	0.200000
5	0.000533	0.000533	-3.273001	0.200000	0.000160	0.000160	-3.795880	0.200000

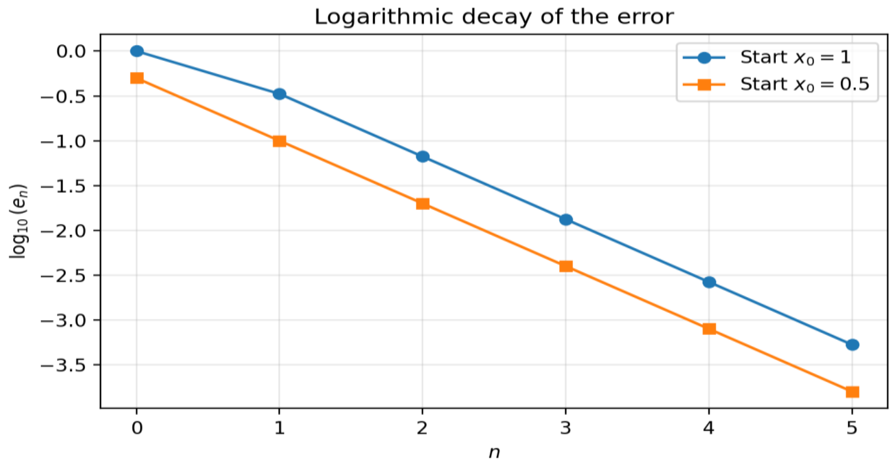

Figure 1. Logarithmic decay of the error $e_n = d(x_n, z)$ *for two different initial points, generated by numerical simulation.*

Remark 4.2. For the start $x_0 = 1$, the first step uses the discontinuous branch $T(1) = 1/3$, hence $r_1 = 1/3$. Afterwards the orbit stays in $[0,1)$ and satisfies $x_{n+1} = x_n / 5$, so $r_n = 1/5$ for all $n \geq 2$. For the start $x_0 = 0.5 \in [0,1)$, one has $r_n = 1/5$ for every $n \geq 1$

5. Conclusion

In this paper, we developed a unified stability framework for fixed point problems associated with Fisher-type contractive mappings. The central conclusion is a global residual estimate, from which Hyers–Ulam stability and well-posedness follow whenever a fixed point exists. If a perturbed mapping has a fixed point and is uniformly close to the original mapping, the same estimate yields data dependence. The resulting constants are explicit and depend only on the Fisher parameter c.

By combining these abstract results with existing fixed point existence theorems, we derived complete stability statements in boundedly compact, T-orbitally compact, and complete metric spaces. An example of a discontinuous mapping, together with numerical illustrations, demonstrated the applicability and generality of the theory.

Future research directions may include extensions to multivalued mappings, generalized metric spaces, or the investigation of stability properties for other rational-type contractive conditions.

References

[1] I. Altun and H. A. Hancer (2019), *Almost Picard operators*, AIP Conference Proceedings, 2183, 060003.

[2] G. V. R. Babu (2004), *Generalization of fixed point theorems relating to the diameter of orbits by using a control function*, Tamkang J. Math., 35(2), 159-168.

[3] V. Berinde (2007), *Iterative Approximation of Fixed Points*, Springer, Berlin.

[4] S. K. Chatterjea (1972), *Fixed-point theorems*, C. R. Acad. Bulgare Sci., 25, 727-730.

[5] Lj. B. Ćirić (1974), *A generalization of Banach's contraction principle*, Proc. Amer. Math. Soc., 45(2), 267-273.

[6] R. E. Edwards (1965), *Functional Analysis: Theory and Applications*, Holt, Rinehart and Winston, New York.

[7] B. Fisher (1978), *A fixed point theorem for compact metric spaces*, Publ. Math. Debrecen, 25, 193-194.

[8] H. Garai, L. K. Dey, T. Senapati (2018), *On Kannan-type contractive mappings*, Numer. Funct. Anal. Optim., 39(13), 1466-1476.

[9] A. Granas and J. Dugundji (2003), *Fixed Point Theory*, Springer-Verlag, New York.

[10] N. H. Hoc, H. K. Anh, L. T. Thu (2022), *Some fixed point theorems for Fisher-type contractive mappings*, Hong Duc University Journal of Science., 12, 23–30.

[11] D. H. Hyers (1941), *On the stability of the linear functional equation*, Proc. Natl. Acad. Sci. U.S.A., 27(4), 222-224.

[12] R. Kannan (1968), Some results on fixed points, Bull. Calcutta Math. Soc., 60, 71-76.

[13] A. Petruşel and I. A. Rus (2007), *Well-posedness of the fixed point problem for multivalued operators*, in: Applied Analysis and Differential Equations, World Scientific, 295-306.

[14] A. Petruşel, I. A. Rus and J.-C. Yao (2007), *Well-posedness in the generalized sense of the fixed point problems for multivalued operators*, Taiwanese J. Math., 11(3), 903-914.

[15] S. Reich and A. J. Zaslavski (2001), *Well-posedness of fixed point problems*, Far East J. Math. Sci., Special Volume (Part III), 393-401.

[16] I. A. Rus (2001), *Generalized Contractions and Applications*, Cluj University Press, Cluj-Napoca.

[17] I. A. Rus (2009), *Remarks on Ulam stability of the operatorial equations*, Fixed Point Theory, 10(2), 305-320.

[18] S. M. Ulam (1960), *Problems in Modern Mathematics*, John Wiley & Sons, New York.